

## Detecting hidden defects on a thin metallic plate

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### Abstract

We present here an algorithm to detect and evaluate a damage on the inaccessible side of a thin metallic plate  $\Omega$  from temperature measurements taken from the opposite side. The algorithm comes from the application of a mathematical tool called "Domain Derivative" which allows us to linearize the original non linear inverse problem for the heat equation

### 1. Introduction

The present work deals with Nondestructive Evaluation methods based on Active Infrared Thermography [5].

A thin metallic plate  $\Omega = (0,1) \times (0,1) \times (0,a)$ , with  $a \ll 1$ , divides an outer aggressive environment from a laboratory. Let  $z=a$  the inaccessible surface of  $\Omega$  in contact with the outside and  $z=0$  the laboratory side. The specimen is assumed homogeneous, i.e. the diffusivity  $\alpha$  of the material is constant. Our purpose is to detect in real time the appearance of a damage when it is as small as possible.

We are able to heat the specimen at  $z=0$  with a heating lamp, inducing a heat flux  $\phi$  through the laboratory side. On the same surface we can take a sequence of temperature maps  $\{T_1, T_2, \dots, T_n\}$  with an infrared camera.

We assume that the only effect of the external aggression to our domain is the loss of an amount of matter; as a consequence  $\Omega$  is modified so that the side  $z=a$  is no longer a plane. Let  $\sigma(x,y)$  the function describing the deviation of the inaccessible side from being a plane, then the damaged domain is now  $\Omega = \{(x, y, z) \mid 0 \leq x, y \leq 1; 0 \leq z \leq a - \sigma(x,y)\}$ .

The temperature in  $\Omega$  satisfies the classical heat equation with exchange boundary conditions of the form, on the surfaces  $z=0$  and  $z=a$ :

$$T_n + \gamma T + \beta \phi = 0 \tag{1}$$

Our goal is to detect  $\sigma$  given  $\phi$  and the data  $\{T_1, T_2, \dots, T_n\}$ .

We know that this problem has a unique solution [4]; however, it is severely noise sensitive and we implement a regularized output least squares, see [2] and [6].

We remark that the use of boundary conditions (1) is highly recommended when the Biot number of the system specimen-environment is large enough, i.e. the exchange parameter  $\gamma$  cannot be neglected. Moreover, if  $\gamma$  is small, the sensitivity of the problem to noise is greatly amplified.

### 2. Our procedure

Our procedure is based on Fourier analysis. We apply Fourier transform in time to the problem for temperature  $T$  and obtain a sequence of Helmholtz type equations for the Fourier coefficients  $W_k$ ,  $k = \dots, -2, -1, 0, 1, 2, \dots$ . In particular, we chose a Helmholtz equation corresponding to  $k=1$  among the others.

Now we linearize the solution in the damaged domain with respect to  $\sigma$ . The first order term of this expansion is the so-called "Domain derivative" a mathematical tool frequently used in inverse problems by now, see for instance [3]. We get now the following problem to solve:

$$\begin{aligned} B_k^2 W_k &= \Delta W_k \\ W_{k,z}(x, y, 0) - \gamma W_k(x, y, 0) &= 0 \\ W_{k,z}(x, y, a) + \gamma W_k(x, y, a) &= \sigma u_k (B_k^2 - \gamma^2) \end{aligned} \tag{2}$$

where  $B_k = \frac{2\pi i k \tau}{\alpha}$  ( $\alpha$  is the diffusivity),  $u_k$  is the Fourier coefficient of temperature in the undamaged domain. We assumed adiabatic boundary conditions on the vertical sides of the plate.

We cannot actually solve the boundary value problem (2). Indeed, we find an integral formulation of it and we reduce our original problem to an infinite linear system of equations of the form (see [1])

$$A_n \sigma_n = \hat{W}_n, n = 0, 1, 2, \dots \quad (3)$$

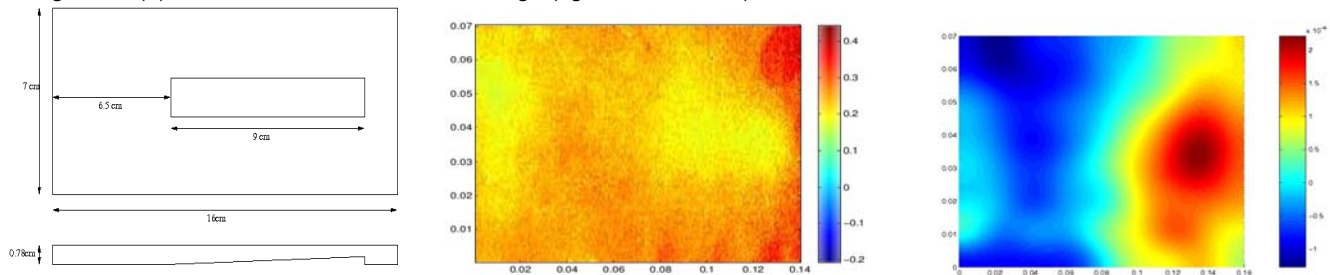
where the coefficients  $A_n$  contains the data of the problem,  $\sigma_n$  are the Fourier coefficients of our unknown function and  $\hat{W}_n$  are the Fourier coefficients of the experimental data. Of course, in our calculations we considered a finite number  $M$  of equations.

Despite the simplicity of the algorithm above we have to point out that it comes from a linearization procedure, i.e. we commit an approximation error of  $O(\sigma^2)$ ; moreover, it is severely noise sensitive since the final error in the solutions grows as  $e^M$ , so we can properly recover only a few number of coefficients  $\sigma_n$ . Finally the solution of the problem becomes harder as the exchange coefficient becomes smaller.

## 2.1. Reconstruction

We present a first attempt to recover the damage from temperature maps.

We performed an experiment using an aluminium slab of 16 cm x 7 cm x 0.78 cm with a damage on the inaccessible side (figure 1 to the left); then we collected temperature maps on the opposite (figure 1 on the center) and using the algorithm (3) we tried to reconstruct the damage (figure 1 to the left)



**Fig. 1.** A sketch of the damaged specimen (left); the phase of the temperature map taken during the experiment, after post-processing of data (center); the reconstruction of the damage using our algorithm (right).

The results are encouraging; indeed the algorithm permits to recover at least the position of the damage. In this stage of our work (which is still in progress) we assumed that some parameters such as the heating flux were constants in space while in real cases this can no longer be true. In further developments we shall consider space variability of such parameters.

## REFERENCES

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